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TAKE-OFF DISTANCE FOR AIRPLANES

By A. Pröll

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By A. Pröll.

With the rapid development of air traffic, the aviation-field problem is constantly increasing in importance. It is not so much the need of good landing places as the indispensable safety in taking off, which often renders difficult the choice of a suitable field. Landings can now be made on small fields, since there are all sorts of devices for shortening the landing run, even for large and swift commercial airplanes. It is only necessary to have a smooth field, which may be slightly sloping and whose dimensions need not exceed 200 m (656 ft.) in all directions. Even without an entirely satisfactory subsoil, such a field will at least serve for emergency landings. The take-off from such a field can be made, however, only under the most favorable conditions and generally not at all.

The conditions for taking off differ greatly from those for landing. With a poorly climbing airplane it is much more difficult to take off over rows of houses or trees, than to alight with the same airplane on the same field. The landing run is almost always shorter than the take-off run in a normal take-off. Only for airplanes with very great climbing ability (i.e., with

* "Die Startstrecke bei Flugzeugen," Zeitschrift für Flugtechnik und Motorluftschiffahrt, August 14, 1926, pp. 316-322.

powerful engines) and a high wing loading, can the take-off run be shorter than the landing run. It has been found, for example, that heavily loaded modern commercial airplanes require a run of 600-700 m (1968-2297 ft.) for taking off, and only about 150 m (492 ft.) for landing. These conditions are a constant source of anxiety for the aviation-field manager of an air-traffic company, who must give them a great deal of attention.

In particular, the question arises as to the possibility of shortening the take-off run. The first phase of flight, when the airplane is gaining headway immediately above the surface of the ground, also belongs, however, to the process of taking off, which can be considered as ended only after the climb has assumed a sufficiently steep angle and the airplane has attained a certain altitude. Sometimes definite numerical stipulations are made (as, for example, the attainment of an altitude of 20 m (65.6 ft.) within 600-700 m (1968-2297 ft.) from the starting line. Such requirements, taken from practical experience, determine the minimum size of a utilizable aviation field for any given commercial airplane.

Artificial devices for shortening the take-off distance are not so effective and not so generally applicable as those for shortening the landing distance, since the retarding effects of the latter are always more simply and easily attained than acceleration in taking off. For instance, the tail skid helps to shorten the landing run, but lengthens the take-off run. Even the new Spanish "Autogiro" requires a take-off run of about 150 m (492 ft.) against a landing run of 20 m (65.6 ft.).

Obviously the first thing to do is to discover when and how the shortest possible take-off can be made with the usual means, i.e., by starting with throttle wide open and with the proper use of the elevator. Can the most favorable effect be obtained by skillful steering and is it worth while to investigate various possibilities?

There are two principal ways in which the total take-off distance (taxying plus hovering plus the first part of the climb) can be reduced to a minimum. These are

1. Taxying and hovering until the maximum speed has been attained close to the ground and then changing to a steep rapid climb.

2. Lifting the airplane from the ground as soon as possible and then climbing at a relatively large angle of attack.

These cases, as well as all other conceivable combinations, can be expressed with a single basic formula, which is derived from the energy equation for rectilinear flight. If G is the weight of the airplane, v the speed and s the take-off distance, then $\frac{G}{g}\, v\, d\, v = d\, s$ (propeller thrust - air resistance - ground friction) (1)

Here we can bring the right side to a quadratic function of the speed in the form $(\alpha - c\ v^2)$ and then find the general solution

$$S = \frac{G}{2\ g\ c}\ \ln\ \frac{a - c\ v_o^2}{a - c\ v^2} \tag{5}$$

by taking the integration between the speed limits v_o (initial speed) and v ("starting speed").*

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* Under the assumption that the propeller thrust follows the equation

$$S = C - B\ v^2 \tag{2}$$

and that the total friction (ground and axle) of the landing gear is expressed by the formula

$$R = \mu\ (G - A) \tag{2a}$$

in which A denotes the coefficient of lift and μ the coefficient of friction, the general expression for the take-off distance is

$$\frac{G}{g}\ v\ d\ v = d\ s \left\{ (C - B\ v^2) - c_w \frac{\gamma}{2g}\ F\ v^2 - \right.$$

$$\left. - \mu \left(G - c_a \frac{\gamma}{2g}\ F\ v^2 \right) \right\} \tag{3}$$

$$s = \frac{G}{g} \int_{v_o}^{v} \frac{v\ d\ v}{(C - \mu\ G) - v^2 \left(B + (c_W - \mu\ c_a) \frac{\gamma}{2g}\ F \right)}$$

$$= \frac{G}{g} \int_{v_o}^{v} \frac{v\ d\ v}{a - c\ v^2} \tag{4}$$

Herein the first part of the take-off distance, the taxying, is

$$a_1 = C - \mu\ G \qquad c_1 = B + (\epsilon_1 - \mu)\ c_a \frac{\gamma}{2g}\ F \tag{4a}$$

wherein the lift-drag ratio ϵ_1 and c_a refer to the angle of attack while taxying and F is the wing area.

After the taxying speed has reached the value v_1 at which the airplane can leave the ground and it has done so through the action of the elevator, it then hovers over the ground until it has attained a speed v_2 sufficient for climbing. This second portion of the take-off distance is computed by the same equations, but with other constants

$$a_2 = C \quad \text{and} \quad c_2 = B + \epsilon_2 \; c_a \frac{F\gamma}{2g} \tag{4b}$$

as soon as $v = v_1$. The take-off distance for the first two portions is then

$$s_1 + s_2 = \frac{G}{2g} \left\{ \frac{1}{c_1} \ln \frac{a_1}{a_1 - c_1 \; v_1^2} + \frac{1}{c_2} \ln \frac{a_2 - c_2 \; v_1^2}{a_2 - c_2 \; v_2^2} \right\} \tag{6}$$

An analytical minimum for this quantity does not exist, in so far as v_2 is given and the most favorable v_1 is sought. It is easily seen, however, from Table I of the example, that it is expedient to choose v_1 as small as possible, i.e., to leave the ground as soon as possible and then, without climbing, to attain the requisite speed while hovering. (An often observed fact thereby is the favorable action of the low-wing monoplane with its short take-off distance, which does not, however, correspond completely with purely theoretical predictions.)

For estimating the third portion of the total take-off distance, namely, the climb to 20 m (65.6 ft.) altitude, the following consideration is of service. If the elevator is deflected sharply upward at the end of the hovering, the airplane, with decreasing speed, describes an upward curve with an initial radius which can be calculated from the centrifugal force

$$\rho_2 = \frac{v_2^2}{g\left(\frac{A}{G} - \frac{1}{F}\right)} \tag{7}$$

in which A is the great lift

$$= c_{a\ max} \frac{\gamma}{2g} v_2^2,$$

to which v_{min} belongs in unaccelerated flight.

ρ increases as the speed decreases and, with v_3, finally assumes the value

$$\rho_3 = \frac{1}{g}\left(\frac{v_3^2\ v_{min}^2}{v_3^2 - v_{min}^2}\right) \tag{7a}$$

At the low altitudes to which this consideration applies, the resulting flight path may be accurately enough regarded as the arc of a circle with the mean radius

$$\rho_m = \frac{1}{2}(\rho_2 + \rho_3) \tag{8}$$

and an arc length of

$$s_3' = \sqrt{2\ \rho_m\ h_1}$$

in which the altitude attained can be calculated from the energy equation

$$G\ h_1 + W_m\ s_3' = \frac{G}{g}\ \frac{v_2^2 - v_3^2}{2} + 75\ N\ \eta\left(\frac{2\ s_3'}{v_2 + v_3}\right)^* \tag{9}$$

* $\frac{2\ s_3'}{v_2 + v_3}$ is the approximate climbing time to h_1.

It is also sufficient to introduce the mean drag W_m which is calculated from

$$W_m = c_{w\,max} \frac{\gamma}{2g} F \left(\frac{v_2 + v_3}{2}^2\right)^2 = \left(\frac{v_2 + v_3}{2\ v_{min}}\right)^2 G \frac{1}{\epsilon_o} . \tag{10}$$

ϵ_o being the lift-drag ratio for the maximum c_a.

From a quadratic equation for

$$h_1 = (\alpha + \beta^2\ \rho_m) - \sqrt{(\alpha + \beta^2\ \rho_m)^2 - \alpha^2}\ * \tag{11}$$

and the corresponding length of the arc, we finally obtain

$$s_3' = \sqrt{2\ h_1\ \rho_m}\ ** \tag{12}$$

To this is then added the climb from h_1 to h with constant (best) vertical speed v_s and with the flight speed v_3. These are both known, however, for every airplane, so that the last portion of the total take-off distance is expressed by

$$(h - h_1)\ \frac{v_3}{v_s} = s_3''$$

The flight paths in this third portion of the take-off appear therefore at various speeds v_2, as represented in Fig. 1, and show that even the choice of v_2 is not without influence on the total take-off distance.

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* Herein for abbreviation is set

$$\left.\begin{aligned} \alpha &= \frac{v_2^2 - v_3^2}{2\ g}, \quad \beta = \left\{W_m - \frac{2\ N\ \eta\ 75}{(v_2 + v_3)}\right\}\frac{1}{G} \\ \rho_m \text{ as above } &= \frac{1}{2g}\left\{\frac{v_2^2\ \ v_{min}^2}{v_2^2 - v_{min}^2} + \frac{v_3^2\ \ v_{min}^2}{v_3^2 - v_{min}^2}\right\} \end{aligned}\right\} \tag{13}$$

** The horizontal projection of s_3', properly coming into the question, is practically equal to the value of the arc, on account of the small angle of climb or the smallness of h_1/ρ_m.

Since, according to what has preceded, v_1 is as small as possible and v_3 likewise appears to be established, there is, therefore, in the choice of v_2, a free space from the lower limits $v_2 = v_1$ or v_3 (taking off and climbing without acceleration) up to $v_2 = v_{max}$ and hence to a pronounced springlike start.

On account of the complicated conditions (equations 6 and 11), it is not expedient to seek an analytical expression for the minimum of the total distance s in terms of v_2. It is better to select the typical cases by means of a numerical example. This is sufficiently illustrated by the subsequent example with the accompanying figures.

The general coefficients in equation (4) must first be more accurately determined, especially the thrust S = C and the flight efficiency (Flugbeiwert) B of the propeller. The air density is also quite important (in equation 4 and in the constant c). The total take-off distance would be considerably increased, if γ should be appreciably diminished by a high temperature near the ground.

The bench thrust of a propeller having a given diameter, pitch, shape, and blade number, can be determined from experimental curves, which have been plotted in large numbers for various propellers. Our case has to do, however, with a general relation between the normal engine power and the prospective propeller thrust. The following results were obtained from the theory of the propeller slip stream.

If w is the reaction velocity of the emerging slip stream, the thrust is then

$$S_o = \frac{\pi D^2}{4} \frac{\gamma}{g} \frac{\omega^2}{2},$$

the engine power

$$L_o = \frac{\pi D^2}{4} \frac{\gamma}{g} \frac{\omega^3}{4}$$

and hence

$$S_o = \sqrt[3]{\frac{\gamma}{2g} \pi D^2 L_o^2} \qquad (14)$$

If we put $L_o \sim 0.8^* \times 75$ N (HP.) $\times 360 = 21600$ kg m/s (14515 ft. lb./sec.) and $\frac{\pi D^2}{4} = 7$ m² (75.3 sq.ft.), then $S_o = 950$ kg (2050 lb.). A 360 HP. engine was taken as the basis of this calculation, the same as used in the subsequent example.

By comparing the bench thrusts of similarly built propellers on various engines, we obtained (from equation 14) the expression

$$S_o : S_o' = \sqrt[3]{N'^2} : \sqrt[3]{N^2} \qquad (14a)$$

A mean value of $S_o \sim 550$ kg (1212.5 lb.)** was established by a large number of bench thrust tests with various propellers on a 160 HP. Daimler engine. Hence S_o may equal 950 kg (2094 lb.) with the 360 HP. engine under similar conditions, whereby we obtain a close confirmation of the above-computed number.

* Because the bench HP. with throttle wide open is only about 0.8 of the maximum HP. in flight.
** Experiments by the writer at the Austrian Army aviation field at Aspern near Vienna.

The revolution speed of a propeller is known to increase from the bench tests up to full speed in the air (in the subsequent example from $n = 1350$ to $n = 1600$ R.P.M., according to the shape of the propeller and the width of the blades). If, in Fig. 2, the thrust curves for $n = 1400$ and $n = 1600$ are plotted as flat parabolas, the course of the propeller thrust, during the start with increasing velocity, is then characterized by the dash-and-dot line which, in turn, can likewise be plotted as a parabola $S = M - K v^2$.

It is possible, moreover, to make up such a propeller-thrust formula from observations of the taxying and of the take-off speed v_1, provided the decisive quantities, lift-drag ratio ϵ, c_a, coefficient of friction and propeller efficiency are known, which, of course, is only approximately true. The calculation is therefore rather unreliable.

The recent announcement of the 1926 South-Germany contest stated that: "The decrease in the propeller thrust and the increase in the air resistance with increase in speed are exactly offset by the decrease in the ground friction due to the lift, so that the acceleration remains constant during the take-off run."

If this were universally true, formula (4) would be considerably simplified, at least for the taxying and we would have

$$S_1 = \frac{G}{2g} \left(\frac{v_1^2}{a_1}\right) \left(\frac{v_1^2}{C - \mu G}\right) \qquad (6a)$$

There would then remain only the possibility of slightly changing the acceleration, and thereby the first portion of the total take-off distance, by varying C (other propellers) within narrow limits. We will test the above statement by numerical values and select for this purpose a commercial airplane with the characteristics

Weight G	3200 kg	7055 lb.
Engine power		360 HP.
Wing area F	62 m²	667.4 sq.ft.
Propeller efficiency η in horizontal flight		65%
Propeller efficiency η in climbing flight		62%
Lift-drag ratio ϵ in horizontal flight		1 : 7.5
Lift-drag ratio ϵ in climbing flight		1 : 6.5

Coefficient of friction μ with reference to the circumference of the wheels:

$$\left.\begin{array}{ll}\text{Axle friction} & 0.006 \\ \text{Ground} \quad " & 0.074\end{array}\right\} \text{together} \quad 0.08$$

The resistance in horizontal flight is then $\frac{3200}{7.5} = 426$ kg (939 lb.) and the speed is

$$v_{horiz.} = \frac{360 \times 75 \times 0.65}{426} = 41.2 \text{ m/s } (135 \text{ ft./sec.}).$$

In unaccelerated climbing flight, $v \sim 28$ m/s (92 ft./sec.) ($c_a \sim 0.9$) can be assumed, which will give a climbing speed of $v_s' \sim 1.2$ m/s (3.9 ft./sec.).

For the maximum angle of attack ($c_{a\ max} = 1.1$), $\epsilon = \frac{1}{6.5}$ and $v = 27.4$ m/s (90 ft./sec.), the minimum speed at which the airplane can take off, with a climbing speed of $v_s \sim 1$ m/s (3.28 ft./sec.). From this is obtained the following constant coefficient

$$a_1 = 950 - 0.08 \times 3200 = 700 \text{ kg (1543 lb.)}.$$

Moreover, in horizontal flight $S = 426 = C - B\ v^2_{max} = 950 - B \times 41.2^2$, according to which $B = 0.31$ and we now have:

$$c = 0.31 + (\epsilon - 0.08)\ c_a \frac{62}{16}.$$

$$\text{For} \quad c_a = 0.5, \quad \epsilon = \frac{1}{7.5}, \quad c = 0.41$$

$$\text{"} \quad c_a = 0.9, \quad \epsilon = \frac{1}{7}, \quad c = 0.524.$$

The first and second portions of the total take-off distance must then be calculated, according to the manner of starting, with the help of equation (4) and the third portion (climb) according to equations (11) and (12).

Table I.

Taxying and hovering with $c_a = 0.49$ with varying v_1 and with $v_2 = 40$ m/s (131 ft./sec.).

Speed v_1	28 91.9	30 98.4	32 105	34 111.5	36 118.1	38 124.7	40 131.2	m/sec. ft./sec.
s_1	240 787	290 951	350 1148	430 1411	545 1788	715 2346	1070 3510	m ft.
s_2	660 2165	622 2041	574 1883	510 1673	422 1385	293 961	0 0	m ft.
$s_1 + s_2$	900 2953	912 2992	924 3031	940 3084	967 3173	1008 3307	1070 3510	m ft.

1. Start at the minimum lift-drag ratio (c_a = 0.41) from 0 to speed v_1. Table I gives the take-off distances for various values of v_1. The common final speed v_2 = 40 m/s (131 ft./sec.) was introduced in line 2 (s_2). As already mentioned, it has been found that the total take-off distance increases as v_1 increases, even though only slightly.

Table II.

Taxying till v_1 = 28 m/s (91.9 ft./sec.), s_1 = 240 m (787 ft.), and c_a = 0.49, constant climb with v_s = 30 m/s (98.4 ft./sec.).

Speed v_2	30 98.4	32 105	34 111.5	36 118.1	38 134.7	40 131.2	m/s ft./sec.
s_2	35 115	86 282	145 476	240 787	368 1207	660 2165	m ft.
$s_1 + s_2$	270 886	326 1070	385 1263	480 1575	608 1995	900 2953	m ft.
Climb to altitude h_1	0 0	4.6 15.1	8.6 28.2	12.6 41.3	17.9 58.7	23.2 76.1 (to 20 m 65.6 ft.)	m ft.
Distance s_3' (to h_1)	0 0	60 197	75 246	90 295	105 344	110 361	m ft.
Further distance s_3'' to h = 20 m (65.6 ft.) (angle of climb 1/25)	500 1640	382 1253	285 935	185 607	52 171	0 0	m ft.
Total take-off dist. $s_1 + s_2 + s_3' + s_3''$	775 2543	763 2520	745 2444	755 2477	765 2510	1010 3314	m ft.

If, therefore, the minimum speed v_1 = 28 m/s (91.9 ft./sec.) is chosen for the moment of take-off, Table II then shows the effect of the various speeds v_2 (for the beginning of the climb) and also gives the values h_1 and s_3 for the third portion of the take-off distance and for the total take-off distance.

2. The method of calculation is perfectly analogous for the take-off with a large angle of attack (c_a = 0.9). The results are given in Table III and Figs. 3-4.

Table III.

Taxying till v_1 = 28 m/s (91.9 ft./sec.), s_1 = 288 m (945 ft.), c_a = 0.9, v_2 variable and v_3 = 30 m/s (98.4 ft./sec.).

Speed v_2	30 98.4	32 105	34 111.5	m/s ft./sec.
s_2	69 226	184 604	490 1608	m ft.
$s_1 + s_2$	357 1171	472 1549	778 2552	m ft.
Climb to altitude h_1	0 0	4.6 15.1	8.6 28.2	m ft.
Distance s_3' (to h_1)	0 0	60 197	75 246	m ft.
Further distance s_3'' (to 20 m - 65.6 ft.)	500 1640	382 1253	285 935	m ft.
Total take-off distance	857 2812	914 2999	1138 3734	m. ft.

From these figures it is manifestly inexpedient to let the airplane "hover" very long, but that the minimum take-off distance, including the first 20 m (65.6 ft.) of climb, is obtainable only by beginning to climb at a speed of 34 m/s (111.5 ft./sec.), just a little in excess of the most favorable climbing speed, here $v \sim 30$ m/s (98.4 ft./sec.).

The coefficient of the ground friction μ, which consists chiefly of rolling resistance (sinking in soft ground), is very important. Taxying up to the minimum take-off speed of 28 m/s (91.9 ft./sec.) requires the values of s_1, as given below for the various values of μ.

$\mu =$	0.08	0.12	0.16	0.2
$s_1 =$	245	260	382	540

An error in the coefficient of friction μ may therefore give a very false result, when the speed has nearly reached its maximum value.

If the condition of constant acceleration during the taxying, as expressed in equation (6a) were true, we would have, for our example with $\mu = 0.08$,

$v_1 =$	28	32	36	40	m/s
	91.9	105	118.1	131.2	ft./sec.
$s_1 =$	180	234	296	366	m
	591	768	971	1201	ft.

with $\mu = 0.1$

$s_1 =$	198	260	330	405	m
	650	853	1083	1329	ft.,

hence considerably smaller values than the ones given in Table I. The difference is largely due to the actually much smaller acceleration at the higher speeds.

In any event, it is much better from the outset to fly "under pressure" and thus to get up speed quicker, than to taxi with a large angle of attack for the purpose of increasing the lift and diminishing the friction.

The longest portion of the take-off distance is the taxying. The most practical way to shorten this seems to be to increase the propeller thrust C, either by a specially designed propeller (with adjustable blades) or by a large excess of engine power during the start.

Such an engine is also very valuable under certain circumstances, but is generally dearly bought by an undesirable increase in the non-paying load. On the contrary, any solution would be desirable, in which the required extra power is supplied from the outside, possibly by storing up energy before the start.

Thus the old Wright airplane was strongly accelerated for a short distance by a falling weight in the "catapult start." Similar devices, employing compressed air, are now used on American battleships, for launching small airplanes. This method has the effect of increasing the propeller thrust and hence of increasing a (or C). This does not, however, elevate the flight path fast enough to meet the above-mentioned requirement of reaching an altitude of 20 m (65.6 ft.) in the shortest possible distance. After the distance s_o of the catapult start, which is much smaller than the previously calculated s_{1+2}, the airplane climbs as before for a distance s and the whole take-off distance is therefore shortened by the ratio $\frac{s_0 + s_3}{s_1 + s_2 + s_3}$ in comparison with the previous take-off distance.*

La Cierva tried another way with his "autogiro" (or windmill airplane). So long as the rotary wings are not mechanically set in rotation at the start, there is hardly any advantage gained. In fact, the take-off distance of this peculiar airplane is still very large in comparison with its very short landing run. There is the possibility, however, of accumulating energy before the start by mechanically setting the wings in rapid rotation. Thereby a considerably increased c_a could be attained due to the greater relative speed of the wings. The attempt has been made, with some success, to produce this rotation by unwinding a rope. It may probably be assumed, however, that the same result will yet be produced by the transmission of engine power.

This example indicates a promising way for shortening the take-off distance, namely, the transmission of energy from without and the accumulation of the same in the airplane before starting. At the present time, this method does not seem applicable to existing airplanes with their fixed wings. In this connection, however, and in spite of its sensational aspect, a pro-

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* In like manner, more or less successful attempts have been made to shorten the landing run by braking with the tail skid and wind flaps. This is the direct counterpart of the catapult start and means a power reduction through external means.

posal, already repeatedly made, gains interest and perhaps sense also.

This concerns the acceleration of a difficultly starting airplane by means of a light towing airplane. It is proposed that the towing airplane (an ordinary but swift unloaded airplane with large reserve climbing capacity) shall fly at a short distance above the starting airplane and shall be coupled to the latter at a given instant. Since the upper airplane is flying faster at first than the already rapidly taxying airplane on the ground, a pull of Z will be exerted on the rope, which will produce both an accelerating and lifting effect on the lower airplane and considerably shorten its take-off run. The towing airplane, on the other hand, will behave, after the coupling, like a suddenly and strongly loaded airplane approaching a stalled condition of flight.

According to the above, the most important problem is to shorten the first two phases of the take-off (taxying and hovering) and it is obvious from Table I that the hardest part of the problem is to attain the high speeds (exceeding 30 m/s (98.4 ft./sec.). Up to this speed the take-off distance is still small (about 270 m - 886 ft.) and no help from the towing airplane is possible or necessary. The latter would then have to give its assistance during the hovering, when it would work in a strongly stalled condition with a very great c_a and a relatively small speed.*

The automatic coupling of the tow line could be effected by a simple device on top of the cabin of the large airplane (if it were a low-wing monoplane) or on the middle of the wing (if it were a biplane or a high-wing monoplane) and would at first exert a pull of zero. The pull could then be regulated by the towing airplane. In order that the latter might not be endangered, an elastic tow line would have to be provided or some device on the towing plane, through which the rope could be reeled in or let out (with an adjustable drag). Lastly, an automatic release of the lower end of the tow line would have to be possible, in case the towing plane should be endangered through too great an increase in the pull. It would also have to be possible for the towing plane, after finishing its task, to drop the tow line before landing.

The following simple computation can be made on the gain to be expected. In the simplest case, the towing plane might fly vertically above and parallel to the large airplane and partially lift it by means of the pull on the tow line. This pull, less the weight of the tow line, would then equal the possible useful load of the unloaded towing airplane at the same speed. This may be found by a simple computation, or graphically by

* We would then use as towing planes special slotted-wing airplanes, or other airplanes with similar characteristics, which could also be used for passenger flights with relatively heavy loads.

means of Everling's lift curve (Fig. 5).*

In our example, we are taking as the basis a towing airplane which weighs 1300 kg (2866 lb.) with a normal load and 2300 kg (5071 lb.) with a maximum load. We can now compute by stages the lifts or lightenings ΔG for speed increases v'' to v' from 26 to 28 m/s (85.3-91.9 ft./sec.), 28 to 30 m/s (91.9-98.4 ft./sec.), etc., and therewith also the reductions in the total take-off distance.

$$\frac{\Delta G}{2\,g\,c} \ln \frac{a - c\,v'^2}{a - c\,v''^2} = \frac{\Delta G}{G}\,(s'' - s')$$

in which c and

$$\ln \frac{a - c\,v'^2}{a - c\,v''^2}$$

remain unchanged with respect to the previous calculation. Also for lifting the airplane up to 20 m (65.6 ft.) altitude a similar computation shows a decided shortening of the take-off distance.** The results are given in Table IV and show that a total saving of about 217 m (712 ft.) or 29% of the total take-off distance can be made. An allowance of 100 kg (220.5 lb.) is thereby made for the weight of the 8 mm (0.315 in.) steel tow line. If an elastic tow line (which would be better under some circumstances) should be used, it would need to have a diameter of at least

* Everling, "Kurvendarstellungen des Fluges." Zeitschrift für Flugtechnik und Motorluftschiffahrt, 1917, p. 34. The example there used is also employed here and concerns a biplane with a 160 HP. Mercedes engine. The squares of v_2 are plotted on the axis of the abscissas and the total lifting forces, or their differences after subtracting the weight of the towing plane (1320 kg - 2910 lb.).

** The towing support is therefore the greatest just at the edge of the aviation field, where the danger is the greatest.

15 mm (0.59 in.) and would weigh about 70 kg (154 lb.). This would, of course, have a much greater drag.

Table IV.

Effect with the towing plane vertically above the other.

a) During the taxying and hovering;

Speed increase (v'' to v')	m/s ft./sec.	26-28 85.3-91.9	28-30 91.9-98.4	30-32 98.4-105	32-34 105-111.5
ΔG less weight of tow line	kg lb.	630 1389	820 1808	890 1962	875 1929
$s''-s'$ (Table II)* + ground friction	m ft.	40 131.2	35 114.8	51 167.3	60 196.8
$\Delta s = \frac{\Delta G}{G}(s'' - s')$	m ft.	8 26.3	9 29.5	14 45.9	16.5 54.1

$\Sigma \Delta s = 47.5$ m (155.8 ft.)

*s here stands for the total distance $s_1 + s_2$.

b) During climb from $v_2 = 34$ m/s (111.5 ft./sec.)
to $v_3 = 30$ " (98.4 ")

		ρ_m	Altitude increase h_1	Dist. s_3'	Distance s_3''	$s_3 = s_3' + s_3''$
Without towing plane (G)	3200 kg 7055 lb.	332 1089	8.6 28.22	75 246	Angle of climb 1/25 285 935	360 m 1181 ft.
With towing plane (G-ΔG)	2315 kg 5104 lb.	168 551	13.36 43.83	66 217	Angle of climb 1/19 124 407	190 m 623 ft.

The shortening $\Delta s_3 = 360-190 = 170$ m (557.7 ft.)

Total shortening of take-off distance 170+47.5 = 217.5 m (713.6 ft.)

Usually, however, the towing plane will fly ahead of the towed plane and exert a steep upward pull, which will both lift and accelerate the latter. In this case, Z_o represents the pull at the upper end of the tow line, including the weight and the drag of the line itself, at the angle θ to the vertical (Fig. 6). Then the weight increase is $Z_o \cos \theta$ and the drag increase is $Z_o \sin \theta$, while G_o is the "clear weight" of the towing plane without the additional load. For the horizontal flight of the latter (engine N_o HP.), we have

$$\frac{75 \, N_o \, \eta_o}{v_o} = \left\{ Z_o \sin \theta + \epsilon_o \, (G_o + Z_o \cos \theta) \right\},$$

in which ϵ_o can be put for the angle of glide and

$$v_o = \sqrt{\frac{G_o + Z_o \cos \theta}{F_o \, c_{ao} \frac{\gamma}{2g}}}$$

for the speed. At the lower end, the tow-line pull Z_u is directed upward at an angle ζ, so that, with the tow-line drag W_s, we have

$$Z_u \cos \zeta + W_s = Z_o \sin \theta .$$

Moreover, $Z_o \cos \theta = Z_u \sin \zeta$ + weight of tow-line.

From these equations, with various angles θ and ζ, we can then calculate all the possible combinations for a given length of tow-line. The quantity C and consequently also a, in our previous formulas, will be increased by the new pull $Z_u \cos \zeta$ and the weight G will be diminished by $Z_u \cos \zeta$. This is quite important when the speed approaches its limit, where $a - c\,v^2$ vanishes. The advantage will therefore be especially great where a is small, due to much friction.

Hence it is expedient, during the last part of the taxying (where $a_1 - c_1\,v^2$ is already very small), for the towing plane, flying in advance, to exert a strong pull at first, while later, at the beginning of the climb (as evident from Table IVb) a greater advantage will be obtained, when the towing plane is almost vertically above the other and the lower airplane is strongly relieved.

By comparative calculations it will not be difficult to determine at what relative positions of the airplanes the best results can be obtained at the various speed stages. Of course these results can also be obtained by systematic tests in actual practice.

The effect and utility of an elastic tow-line can also be approximately computed. Both airplanes would then, under certain conditions, fall into oscillations, which might possibly be utilized for shortening the take-off distance (at least the taxying portion).

It may be of some use to make a thorough study of the conditions, structural details and safety precautions relating to the take-off helps just considered. This would perhaps afford some prospect of success to the seemingly venturesome suggestion, in spite of the obviously great practical difficulties involved.

Translation by Dwight M. Miner,
National Advisory Committee for Aeronautics.

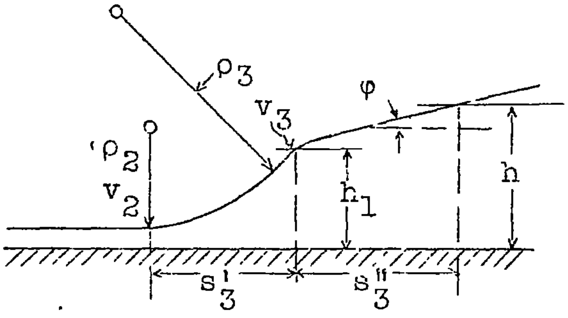

Fig.1

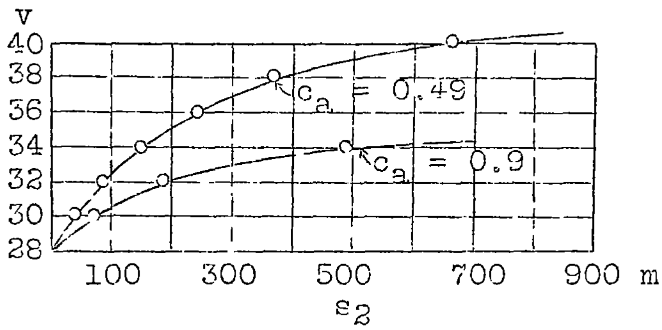

Fig.3

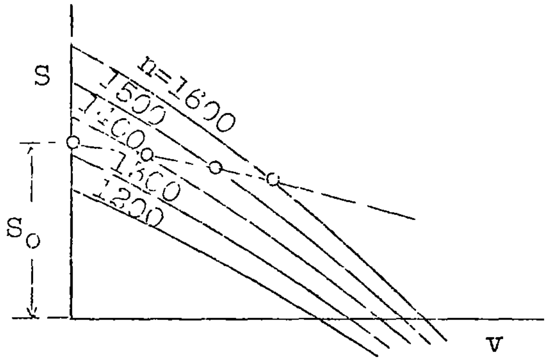

Fig.2

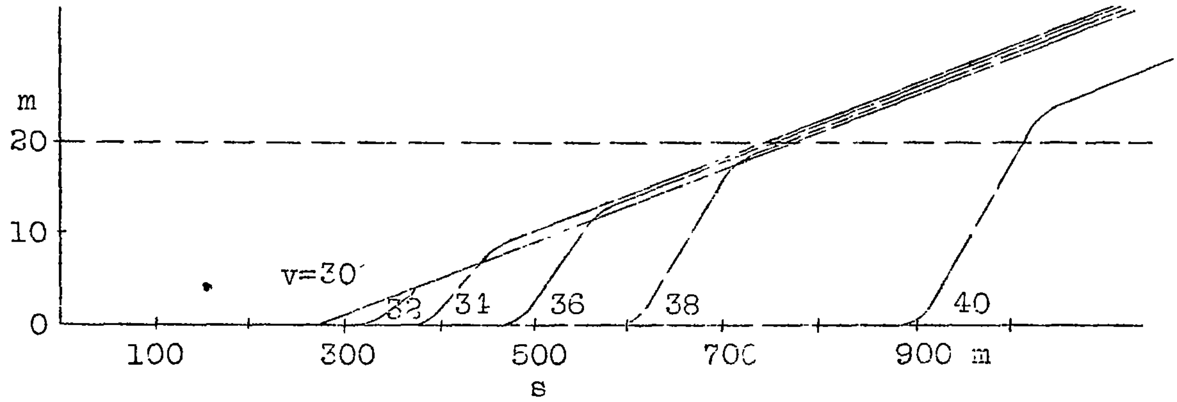

Fig.4

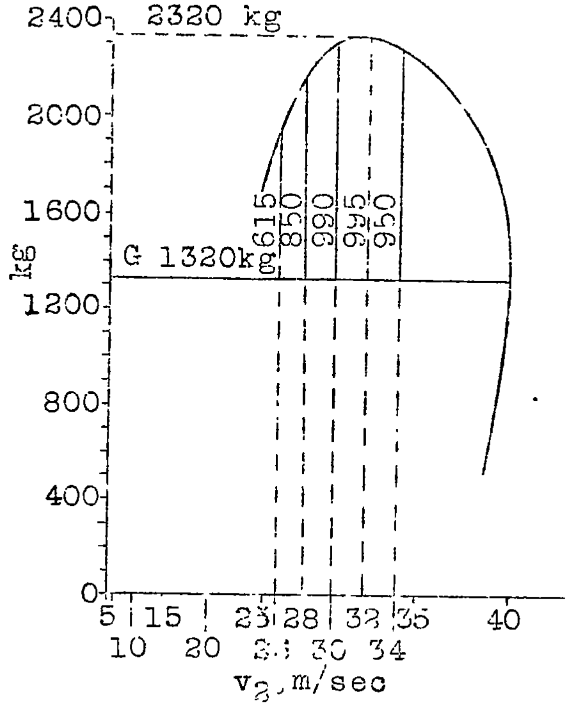
2400
2320 kg
2000
1600
1200
800
400
0
kg
G 1320kg
615
850
990
945
950
5
10
15
20
25
28
30
32
34
35
40
v_2, m/sec

Fig.5

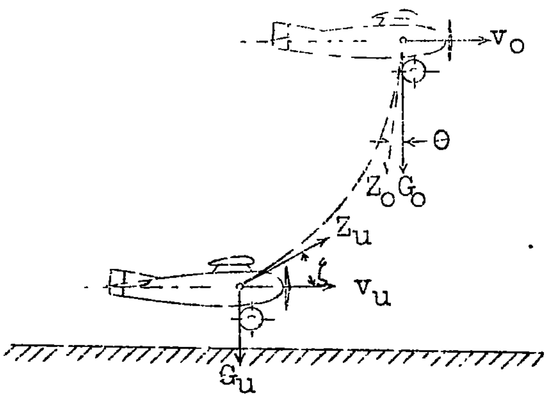
v_o
Θ
Z_o
G_o
Z_u
v_u
G_u

Fig.6